8. S. G. Mason, "Orthokinetic phenomena in disperse systems," J. Colloid Interface Sci., 58, No. 7, 275-285 (1977).
9. P. Bagchi, "Test for particle size dependence in the theory of slow flocculation at a secondary minimum," Colloid Polym. Sci., 254, No. 10, 890-894 (1976).
10. G. R. Wiese and T. W. Healy, "Effect of particle size on colloid stability," Trans. Faraday Soc., 66, No. 3, 490-499 (1970).
11. A.-H. Nayfeh, Perturbation Methods, Wiley, New York (1973).
12. E. A. Novikov, "Turbulent diffusion in a stream with a transverse velocity gradient," Prikl. Mat. Mekh., 22, No. 3, 412-414 (1958).
13. D. E. Elrick, "Source functions for diffusion in uniform shear flow," Aust. J. Phys., 15, No. 3, 283-288 (1962).
14. N. A. Frankel and A. Acrivos, "Heat and mass transfer from small spheres and cylinders freely suspended in shear flow," Phys. Fluids, 11, No. 9, 1913-1918 (1968).
15. G. K. Batchelor, "Mass transfer from a particle suspended in fluid with a steady linear ambient velocity distribution," J. Fluid Mech., 95, Part 2, 369-400 (1979).
16. R. T. Foister and T. G. M. van de Ven," Diffusion of Brownian particles in shear flows," J. Fluid Mech., 96, Part 1, 105-132 (1980).

---

# CONTROLLING DEPOSITION OF A SUBSTANCE ONTO THE INNER SURFACE OF A CYLINDRICAL CHANNEL

V. V. Levdanskii

UDC 536.422.4

The problem of material deposition on the inner surface of a channel with free-molecular gas flow is considered.

Deposition of material from the gaseous phase is one of the basic methods of producing thin layers of material [1]. This method permits production of layers with thickness varying according to a specified rule. At sufficiently low pressures of the vapors of the material to be deposited, a free-molecular flow regime is created in the gas, and the thickness of the layer deposited is dependent only on system geometry, the molecular flux incident on the surface, and the interaction conditions between gas molecules and solid surface.

At the present time external problems have been studied thoroughly. In these cases, the deposition process can be controlled either by changing the position of the material source relative to the substrate or by changing the geometric parameters of the source itself. More complicated and less well studied are internal deposition problems. Thus, for example, in the case of deposition of a thin layer of material on the inner surface of a cylindrical channel, where the source of material to be deposited lies outside the channel, the deposition process can be controlled only by varying the gas pressure at the channel ends and the temperature distribution along the channel.

In the present study we will examine questions of deposition of material layers on the inner surface of a cylindrical channel for a specified material deposition rule along the channel at arbitrary values of the molecular condensation (adhesion) coefficient $\alpha$.

Let it be required to deposit a thin layer of condensate along the inner surface of a cylindrical channel, with the layer thickness varying along the channel length by a specified rule. Naturally, the rule for change in layer thickness along the channel must determine the resulting molecular flux into the condensate.

We will make some simplifying assumptions. We assume that at the start of the process the entire inner surface is coated by a layer of condensate (i.e., we will not consider the process of condensate formation on the bare substrate). The condensate formed is assumed solid. We will consider the problem in the quasistationary approximation. We assume that the growing

---

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 4, pp. 573-577, October, 1982. Original article submitted July 13, 1981.

0022-0841/82/4304-1090$07.50

condensate layer has no effect on the probability of molecular motion from one surface element to another, i.e., the condensate layer thickness during the course of the entire process is assumed to be much less than the channel radius (in the conclusion we will consider a case where this assumption is not required). The rule for change in layer thickness with time has the form $n dh/dt = J$, where J is the resultant molecular flux into the condensate. Our problem consists of finding the conditions under which a specified value of J will be achieved. For a cylindrical channel the flux J can be written in the form

$$J = \alpha\left[\int_0^1 (j + I) K_1(|x - x'|)\,dx' + N_0 K(x) + N_1 K(1 - x)\right] - j, \tag{1}$$

$$I = (1 - \alpha)\left[\int_0^1 (j + I) K_1(|x - x'|)\,dx' + N_0 K(x) + N_1 K(1 - x)\right], \tag{2}$$

where x is a dimensionless coordinate ($x = X/L$); $K_1$, K characterize the probability of molecular motion from one surface element to another [2]; $N_0$, $N_1$ are the flux densities of molecules entering the channels through its ends at $x = 0$, $x = 1$; I is the flux density of reflected molecules. The term in square brackets in Eqs. (1), (2) defines the flux of molecules incident on the portion of the capillary surface in the vicinity of the point x, which consists of molecules which have arrived from the remaining surface after evaporation, or after elastic (but diffuse) reflection, and also molecules which have arrived directly at the given point after entry into the channel from outside (with no beforehand collisions with the wall). The quantity j can be defined in the following manner [3, 4]:

$$j = A \exp\left\{-\frac{Q}{kT}\right\}. \tag{3}$$

The preexponential factor A depends on the model used in deriving Eq. (3). Thus, in the simplest case it is equal to $n(kT/2\pi m)^{1/2}$ [3]. Other (more accurate) expressions for the preexponential factor are presented in [4].

It should be noted that kinetic equation (3) does not contain the evaporation coefficient, which is introduced as a correction factor with use of Langmuir's formula [4] for the flux of evaporating molecules. The effect of impurity particles adsorbed on the phase front, which change the evaporation rate, should be considered in the kinetic relationships by means of a change in the heat of evaporation and the preexponential factor. When Langmuir's expression is used the effect of a different type of impurity is considered in the change in evaporation coefficient.

Combining Eqs. (1), (2), we obtain

$$J + y = \int_0^1 y K_1(|x - x'|)\,dx' + N_0 K(x) + N_1 K(1 - x), \tag{4}$$

where $y = j + I$.

Approximating $K_1$, K by exponential expressions [2], differentiating Eq. (4) twice, and combining the expression obtained with the original equation, we obtain the following differential equation for y:

$$\frac{\partial^2 y}{dx^2} = l^2 J - \frac{d^2 J}{dx^2}, \tag{5}$$

the solution of which has the general form

$$y = l^2 \int\left(\int J(x)\,dx\right)dx - J + ax + b. \tag{6}$$

The constants $a$ and b are found by substitution of Eq. (6) in Eq. (4) and equating the coefficients of $\exp\{-lx\}$, $\exp\{lx\}$.

Further, substituting the y value thus found in Eq. (1), we find the relationship between J and j, which permits finding the distribution T(x) required for the given function J(x).

As examples we will consider cases where the condensate film thickness must vary with coordinates by exponential and linear laws or not vary at all (uniform deposition), i.e., $J_1 = D\exp\{\beta x\}$, $J_2 = Bx + C$, $J_3 = \mathrm{const} = C$.

The solution for y in these cases, according to Eq. (6), can be written in the form

$$y_1 = D\left(\frac{l^2}{\beta^2} - 1\right)\exp\{\beta x\} + a_1 x + b_1, \tag{7}$$

$$y_2 = l^2\left(\frac{Bx^3}{6} + \frac{Cx^2}{2}\right) + a_2 x + b_2. \tag{8}$$

The expression for $y_3$ in the case of uniform deposition follows from Eq. (8) at B = 0. The coefficients $a_i$, $b_i$ are obtained by substitution of the solution for y in the original integral equation:

$$a_1 = \frac{l}{l+2}\left\{N_1 - N_0 + D\frac{(l-\beta)\,l}{\beta^2}\,[1+\exp\{\beta\}]\right\},$$

$$b_1 = N_0 - \frac{Dl\,(l-\beta)}{\beta^2} + \frac{1}{l+2}\left\{N_1 - N_0 + \frac{Dl\,(l-\beta)}{\beta^2}\,[1+\exp\{\beta\}]\right\},$$

$$a_2 = \frac{1}{6\,(l+2)}\,[6l\,(N_1 - N_0) - 3Cl^2(l+2) - 12B - Bl^3 - 3Bl^2 - 6Bl],$$

$$b_2 = N_0 + \frac{B}{l} - C + \frac{6l\,(N_1 - N_0) - 3Cl^2(l+2) - 12B - Bl^3 - 3Bl^2 - 6Bl}{6l\,(l+2)},$$

and $a_3$, $b_3$ are obtained from $a_2$, $b_2$ by setting B = 0.

We note that in a similar manner we can obtain a solution for y for other forms of the function J(x), e.g., in the case where J(x) is a periodic function of the coordinate.

Having obtained a solution for y, from Eq. (3) we obtain the temperature distribution which must be created for the given layer deposition rule:

$$T = Q/(k\ln(A/j)). \tag{9}$$

We will consider in greater detail the case J = const (uniform deposition). In this case, for j we obtain from Eqs. (1), (2)

$$j = \alpha\left(\frac{Cl^2x^2}{2} + a_3 x + b_3\right) - C, \tag{10}$$

which at $\alpha = 1$ coincides with the result of [5]. From Eqs. (9), (10) in the case $N_0 = N_1$ we have

$$T = \frac{Q}{k\ln\left\{\frac{\alpha C\,[N_0/C - l/2 - l^2/2\cdot x\,(1-x)] - C}{A}\right\}^{-1}}. \tag{11}$$

It is evident from Eq. (11) that with decrease in the condensation coefficient T decreases. Also following from Eq. (11) is a limitation on the maximum rate of material deposition, which depends on the ratio of channel length to channel radius. The greater this value, the lower the deposition rate can be.

We note that in the final case (uniform deposition) the assumption of smallness of the layer thickness in comparison to channel radius can be removed in principle, inasmuch as the channel geometry does not change, only the radius varying.

The law for change in radius with time has the form

$$r = r_0 - h(t). \tag{12}$$

From Eqs. (11), (12), it is evident that to realize a uniform deposition in the general case the temperature must vary in a specific manner with time.

## NOTATION

L, r, channel length and radius; $l = L/r$; t, time; n, molecular density in condensate; m, molecular mass; $\alpha$, condensation coefficient; Q, molecular evaporation energy; T, temperature; k, Boltzmann constant; $N_0$, $N_1$, molecular flux densities entering channel at each end.

## LITERATURE CITED

1. Deposition from the Gas Phase [in Russian], Atomizdat, Moscow (1970).
2. A. I. Ivanovskii, "Some questions on the interaction of a measurement cavity with a rarefied gas flow," Tr. TsAO, No. 56, 49-96 (1964).
3. J. Frenkel, Kinetic Theory of Liquids, Peter Smith.
4. S. M. Scala and G. L. Vidale, "Vaporization processes in the hypersonic laminar boundary layer," Int. J. Heat Mass Transfer, 1, No. 1, 4-22 (1960).
5. V. V. Levdanskii, "The effect of temperature distribution on the process of condensation in a capillary," in: Questions on the Theory of Transfer Processes [in Russian], ITMO, Minsk (1977), pp. 84-86.

---

# SUPERSONIC RADIATION WAVES WITH MOTION OF PLASMA

I. V. Nemchinov, M. P. Popova,
and L. P. Shubadeeva

UDC 533.95

A numerical solution is obtained to the two-dimensional radiation-gasdynamics problem of plasma formation under laser action.

Laser radiation acting on a target produces absorption waves which travel in the opposite direction to the laser beam. The leading role in the wave propagation process can be played here by any of the various mechanisms, heat conduction (slow luminous combustion), hydrodynamics (luminous detonation), or spontaneous thermal radiation of plasma (subsonic and supersonic radiation waves), depending on the density and the composition of the gaseous medium as well as on the flux density and the wavelength of laser radiation and on the duration of the laser pulse [1]. In supersonic radiation waves [1-3] the plasma, especially near their fronts, remains almost stationary and its density remains near its initial level. Such a pattern has been confirmed by one-dimensional calculations [2] as well as by experiments [3, 4]. This agreement between theory and experiment was subsequently utilized as the basis for calculating two-dimensional supersonic radiation waves [5]. Assuming a constant plasma density made it possible to treat the equations of radiation transfer simultaneously with only one energy equation, without the need to resort to the complete system of equations of gasdynamics.

The role of gasdynamic processes is not always minor, however, even when the propagation of wave fronts is effected by the radiation mechanism. The plasma can move intensely far away from the fronts of radiation waves. Furthermore, as the radiation flux density gradually decreases and transition to luminous detonation occurs, the role of plasma motion becomes increasingly significant. Accordingly, the method of calculations in [5] was extended to the complete system of equations of radiation gasdynamics in the axisymmetric formulation

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{v}) = 0, \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \operatorname{div}(\rho u \vec{v}) + \frac{\partial p}{\partial z} = 0, \tag{2}$$

$$\frac{\partial \rho w}{\partial t} + \operatorname{div}(\rho w \vec{v}) + \frac{\partial p}{\partial r} = 0, \tag{3}$$

$$\frac{\partial \rho e}{\partial t} + \operatorname{div}(\rho e \vec{v} + p \vec{v} + \vec{q}) = 0, \tag{4}$$

$$\frac{\partial I_\nu}{\partial s} = -k_\nu (I_\nu - B_\nu),\ \vec{q} = \int\limits_{4\pi} \int\limits_0^\infty I_\nu d\nu \vec{\Omega} d\Omega, \tag{5}$$

$$e = e(T,\ \rho),\ p = p(T,\ \rho). \tag{6}$$

---

O. J. Schmidt Institute of Terrestrial Physics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 4, pp. 577-581, October, 1982. Original article submitted July 2, 1981.

0022-0841/82/4304-1093$07.50